The electromagnetic component of large air showers

1970 J. Phys. A: Gen. Phys. 3 284

(http://iopscience.iop.org/0022-3689/3/3/015)

The electromagnetic component of large air showers

E. W. KELLERMANN and L. TOWERS
Department of Physics, University of Leeds
MS. received 22*nd January* 1970

Abstract. Nine scintillators each of area $1{\cdot}1\ m^2$ and $7{\cdot}8\ g\ cm^{-2}$ thickness have been placed 500 m from the centre of the Haverah Park array. Comparison of the signals from these detectors with the Haverah Park Čerenkov tanks near them, and analysis of the energy-loss spectra measured by the scintillators at larger distances from the cores of showers of nominal size 2×10^7 particles, has yielded the lateral distributions of number and energy flux of the incident electromagnetic component from 200 to 1100 m. It is found that the energy flux of this component, mainly photons, varies over this distance from 1000 MeV m^{-2} to of the order of 1 MeV m^{-2} and the photon number from 1000 m^{-2} to of the order of 1 m^{-2}. Spectra of the photons incident at the larger core distances can be obtained in fair approximation.

1. Introduction

Air shower experiments today employ energy-sensitive rather than particle-number detectors, and hence ideas about shower size and particle numbers are being replaced by estimates of shower energies. Particular consideration is given to the partition of this energy amongst the various shower components and the location of the components in the shower. Only by such estimates can shower development and its relation to the nature and energy of the primary particles be properly understood.

It was established by our previous experiment (Dufresne *et al.* 1965) that, even at very large distances from the shower axis in the plane of detection, muons are not the only carriers of energy but that, e.g. at 500 m distance from the shower core, they are still accompanied by a measurable electromagnetic shower component. It is important, therefore, to observe this component in some detail, and to measure the energy flux carried by it. Near the shower core, cascade processes are predominant in the electromagnetic component, but for larger distances from the core the Compton effect is the dominant process in its attenuation. Both these processes characteristically affect recognition by the various detecting devices, and the understanding of the performance of the detectors is a precondition for arriving at an estimate of the energy and lateral distribution of the electron–photon component. For the case of small shower arrays these problems have been discussed by Allan *et al.* (1960, 1962) who have given a bibliography of earlier work. The existence of the Haverah Park array (see Tennent 1967) has enabled us, however, to extend experiments on the electromagnetic component to very much larger distances from the core.

In our work, scintillators were used which are thin ($7{\cdot}8\ g\ cm^{-2}$) compared with the deep Haverah Park Čerenkov water-tank detectors ($120\ g\ cm^{-2}$). They are similar to the scintillators which have been frequently employed by other workers. In our first experiment (Dufresne *et al.* 1965) three such scintillators, each of area $1{\cdot}1\ m^2$, were placed near the central hut of the Haverah Park array. Signals from them and from the Čerenkov tanks nearby were compared for distances of 200 to 500 m from the cores of showers with which they were associated. At these radial distances the average energy of electrons was found to be low enough for photons to be significantly more penetrating than the electron flux in equilibrium with them, and for photons accordingly to be more numerous than electrons. The mean photon energy was found to decrease from about 20 MeV to a few MeV over these radial distances, and the response of the Čerenkov tanks per incident electron at a few hundred metres from the core was of the order of 100 MeV, arising from about 10 photons accompanying each electron. Since the energy flux of the electromagnetic component is by no means restricted to these radial distances, we decided to improve on our preliminary results principally by making fuller use of the extent of the Haverah Park array and extending our work to larger core distances.

2. The scintillators' response to air showers

Nine scintillators, each of the same area and thickness as before, were placed in one of the outer huts of the array, 500 m from its centre. This hut also contained, during some of the running time of the scintillator experiment, the muon detectors used by Allan *et al.* (1968), the data of which we used from time to time for direct comparison, in addition to the normal complement of the Haverah Park Čerenkov detectors of 34 m^2 area. The scintillators were again calibrated by a narrow-angle muon telescope using the value of the mean of the distribution. The necessary checks for uniformity of the signals over the scintillator area and of the stability of the electronics, as well as a thorough evaluation of systematic errors, were made as in our previous work (cf. Dufresne 1968).

Pulses from the scintillators were displayed individually and photographed on a multibeam oscilloscope, the time base of which was fired by a master trigger pulse of the main Haverah Park array, and their images were measured in projection. Corrections were made for slight differences of the detector signals arising from the electronic processing.

The most direct and consistent view of scintillator signals is to regard them as reflecting the energy losses of the incident radiation. One can, of course, interpret the signals as particle densities in terms of 'equivalent muons', i.e. by expressing the signals in terms of the energy loss which a muon fully penetrating the detector would suffer—an approach which is helpful, within limits, provided one avoids statistical analysis in terms of such particles. This can mask the physical processes, both in the showers and in the detectors, which must be fully understood if the analysis is to be carried out correctly. We found it essential not only to measure the pulse heights of the detectors' signals and interpret them as 'equivalent particle densities', but to draw the distributions in energy of the signals. Addition of the signals of all the scintillators in one shower yields the equivalent particle densities incident on the detectors. Diagrams of the added pulse-height distributions of the nine individual scintillators appreciably improved our statistics and materially assisted our understanding of the scintillator response to photons at large distances from the shower core, as will be shown below.

With the help of the information available from the Haverah Park 500 m array, all our observed events were classified according to the zenith angle of the shower, the distance of the recorded event from the shower core, and the energy of the primary particle (or the 'size' of the shower). In the present analysis we used only showers with zenith angles less than or equal to 30°. The total number of individual scintillator detector responses used for finding the pulse-height distribution was 4079, distributed as shown in table 1.

The number of detector responses is not necessarily nine times that of the showers, taking into account occasional detector failure.

Table 1

Detector distance from axis (m)	300–399	400–599	600–799	800–999	$\geqslant 1000$
Number of detector responses	462	2070	940	454	153
Number of showers	52	230	105	51	17
Number of zeros	9	190	270	170	47

3. Comparative response of scintillators and Čerenkov tanks

At core distances greater than 200 m, which are large compared with the characteristic scattering length, it can be assumed that electrons are secondary to photons in the electromagnetic component incident on the detectors. The scintillators and the Haverah Park Čerenkov tank detectors differ in their response to the energy of the incident electromagnetic radiation and also in the variation of their respective responses with core distance, since the mean energy of the incident radiation varies with distance from the core. It is because the responses of the two kinds of detectors differ and are in fact complementary that it is possible by combining them to determine the lateral distributions of the electromagnetic component and of its energy flux over the core distances from 200 to 1100 m, if a knowledge of the muon distribution is assumed.

It was found necessary to consider the signals of the scintillators and of the Čerenkov tanks near them in two distance intervals: those arising from events when the shower core was less than 500 m from the detectors and those with greater than 500 m core distance. For the size of showers which are preferentially selected by the Haverah Park array, the electromagnetic component at about 200 m from the core consists of electrons and photons which can have an energy of up to 20 MeV. These cannot be absorbed completely in the fairly thin scintillators, and therefore the energy loss measured by the scintillators gives an inadequate description of these more energetic particles, whereas the deep Čerenkov tanks can measure the total energy lost—mainly by Compton scattering and, to a lesser extent, by pair production. On the other hand, beyond 500 m core distance, the energy of the photons will be shown to fall to 1 MeV and less. Here, where Compton processes are almost entirely responsible for the attenuation of the electromagnetic component, the scintillators will absorb most of the photon flux, and it is the Čerenkov tanks which are inefficient in measuring the low-energy photon component; they, in fact, hardly register the effect of photons with energy of the order of 1 MeV, since most of the energy is dissipated by electrons below the Čerenkov threshold (Tennent 1967). For these reasons it is meaningful to compare 'density' responses in terms of equivalent muons, registered by the two kinds of detectors up to core distances of 500 m as in our previous work (Dufresne *et al.* 1965), but not beyond.

If D_c is the total equivalent particle density measured by the tanks and D_μ the muon density then, for an energy loss of 2 MeV/g cm^{-2} and the tank thickness of 120 g cm^{-2},

$$\Delta_c = 240(D_c - D_\mu)$$

is the energy loss of the electromagnetic shower component in MeV. This energy loss is measured by the tanks up to distances (about 500 m from the core) where the photon energies are larger than 1–2 MeV so that they are still recognized by the tanks. At these distances, the scintillators, only about 8 g cm^{-2} thick, indicate the density D_s of the incident electrons; but at larger core distances, where the energy of the photons falls to a fraction of 1 MeV, they are measuring more and more accurately, with decreasing photon energy, the energy loss of the electromagnetic component

$$\Delta_s = 16(D_s - D_\mu) \qquad \text{MeV}.$$

A simple calculation (Lillicrap 1963) shows that the track length, i.e. the energy loss ϵ per incident electron arising from the electromagnetic shower component incident on the detector, is, in units of the detector depth t_0,

$$\epsilon = \frac{\lambda}{t_0}\left\{1 - \exp\left(-\frac{t_0}{\lambda}\right)\right\}.$$

Here λ, the attenuation length of the photons, which can be assumed to be the same in water, air and the scintillator material, and t_0, the vertical depth of the detector, are both in g cm^{-2}. This expression depends on the photon energy only in as far as the photon attenuation length λ is energy dependent. Evans (1955) has shown that λ varies from a near-constant value of 60 g cm^{-2} for photons of energies between 100 MeV and a few MeV, to a value of about 20 g cm^{-2} for photons of energies of a fraction of 1 MeV. For the thin scintillators, $t_0 \simeq 8$ g cm^{-2}, the energy loss is almost constant and near 1, varying in fact by only about 10% over the total photon energy range observed. Since the number of incident electrons per square metre is

$$\frac{\Delta_s}{\epsilon_s} = \frac{\Delta_c}{\epsilon_c}$$

the ratio Δ_c/Δ_s (figure 1) yields ϵ_c, the track length per incident electron arising from the

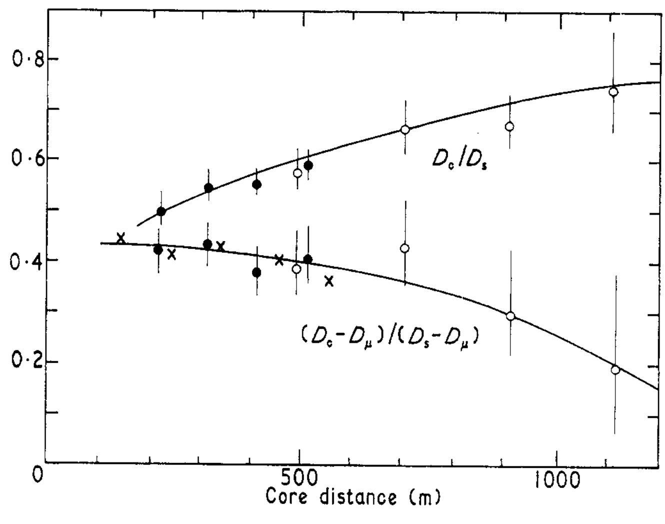

Figure 1. D_c/D_s and $(D_c - D_\mu)/(D_s - D_\mu)$ as a function of core distance.

interactions of photons and electrons incident on the Čerenkov tanks. Since ϵ_c varies much more rapidly with λ and with energy than the corresponding track length ϵ_s for the scintillators, it follows that Δ_c/Δ_s is a measure of the mean energy of the incident photons. Hence, because Δ_c up to 500 m measures the total energy of the incident photons, one obtains by division N_0, the number of incident photons. The usefulness of this method of analysis by density comparison terminates at about 500 m core distance, since beyond this distance the tanks are sensitive almost exclusively to the muon component and the mean photon energies are too low to be recognizable by the Čerenkov detectors, so that $D_c - D_\mu \simeq 0$.

Conversely, from this distance onward, the scintillators begin to measure more accurately the energy of the photons which now is low enough to be recognized even by these thin detectors. Hence, from about 500 m core distance, Δ_s, and not Δ_c, provides an estimate of the photon energy.

The mean energy can no longer be estimated here in the same way as for the nearer core distances. On the other hand, we show below that at the larger distances the number of incident photons N_0 can be obtained by an analysis of the energy-loss spectrum as measured by the scintillators, so that the lateral distribution of the incident photons and hence of their mean energy can be determined for the larger distances as well, but in a different way.

4. Separation of muon contribution and normalization of density data

All density (or energy) determinations of the electromagnetic component presuppose the availability of fairly accurate muon data, making it possible to separate the muon contribution from the detector signals. The muon data reported by Allan *et al.* (1968) were available to us and were used in our analysis. They do not include muons of energy less than 300 MeV, but after considering carefully the muon measurements and distribution functions proposed by other workers (Greisen 1960, Earnshaw

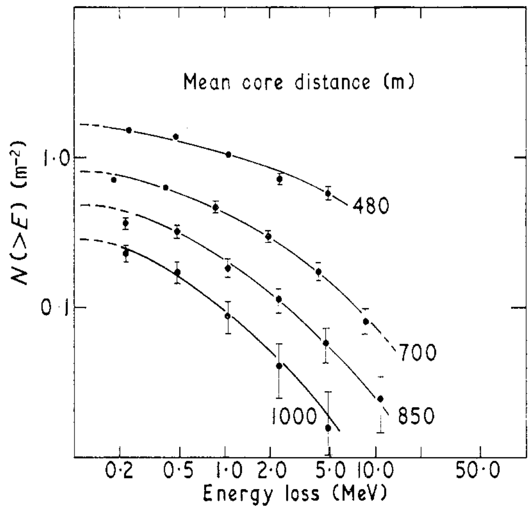

Figure 2. Integral energy-loss spectra per incident particle.

et al. 1968, Vernov *et al.* 1968) and the statistical errors, we think that the error due to the omission of the low-energy muons is contained in the overall error of the values of D_μ, which is quite large. This large error is of course reflected, and leads to at least as large an error in our estimate of $D_s - D_\mu \propto \Delta_s$, i.e. of the total photon energy flux at the larger distances. However, a direct estimate of this energy from the energy-loss spectrum measured by the scintillators, by subtracting the muon contribution from this spectrum as explained below, convinces us that the errors are not larger than given in figure 2.

All equivalent densities were normalized to the same shower size, $N_s = 2 \times 10^7$, corresponding to an energy of the primary of about $1\cdot5 \times 10^{17}$ eV. For distances of 200 to 500 m we can write

$$\frac{D_c - D_\mu}{D_s - D_\mu} = \frac{D_c/D_s - D_\mu/D_s}{1 - D_\mu/D_s}$$

and average the observed ratios over the narrow size interval of about 1×10^7 to 3×10^7. Their spread is small enough to justify this procedure and, at the smaller core distances, the statistics are sufficient even if the size interval is thus restricted. At the larger distances (> 500 m) all density data available were used to determine $D_s - D_\mu$, and here the muon densities D_μ were normalized by assuming a $N_s^{0\cdot75}$ dependence, whereas, for the normalization of the scintillator densities D_s representing the densities of the incident particles, a dependence proportional to N_s was assumed. The distances given in the graphs are the means of smaller distance intervals. All data refer to showers with zenith angles 0–30°.

5. The energy-loss spectrum recorded by the scintillators

The pulse-height distributions recorded by the scintillators, when plotted as histograms and grouped according to core distances, reveal two distinct peaks: one, corresponding to an energy loss of about 16 MeV, is due to particles fully penetrating the detectors' thickness, i.e. muons or such high-energy electrons or photons as may still exist at these core distances in the shower; the other is associated with the bulk of the electromagnetic component. To facilitate clarity of the graphical representation we have drawn histograms for only three of these groups. They are given in frequency of pulse height (in MeV) per detector event (figure 3).

The striking feature of the diagrams is the systematic change in the relative heights of the two peaks with core distance. The division of the histograms into energy intervals masks what is essentially a continuous spectrum of pulse heights compatible with a broad incident spectrum of the electromagnetic component, on which is superimposed a narrower band due to muons near 16 MeV. It is worth noting that the major effect of variation of shower size (or primary energy), within the range of the showers corresponding to the variation with zenith angle considered here, is on the multiplicity $\bar{n}$ of the incident particles. After allowing for this, the energy-loss distribution per incident particle is affected little by the variations in energy of the primary within the range observed here.

All histograms show a significant frequency of zero-height events in which no pulse was recorded ('zero-pulses') and the frequency of these 'zeros' was found to vary from 2 to 31% of all events, with increasing core distances, which clearly reflects the thinning out of the electromagnetic component with core distance. We examined how far these 'zeros' could be instrumental by a progressive lowering of the discrimination bias of the detectors and found that the number of zeros was not greatly affected even by accepting pulses of less than 0·25 MeV. (The number of zeros is given also in table 1).

For the subtraction of the muon contribution from the histograms it is necessary to estimate the pulse-height contribution due to a given density of incident muons. This estimate involves a further error so that the remainder of the histograms representing the electromagnetic component cannot be accurate at the high-energy end to better than $\pm 30\%$.

Conversely, if one assumes that at large core distances practically no high-energy electromagnetic particles are present, one can attribute the majority of pulses near the 10–15 MeV region in the histograms, obtained for, say, 800 m and greater than 1000 m, to muons, estimate from the measured peak the number of muons at these distances, and compare them with the measurements by Allan *et al.* (1968). A similar estimate can be made by analysing showers incident with large zenith angles. We find that at these large distances the muon values obtained by Allan *et al.* (1968) are about 15% below those estimated by us in this way.

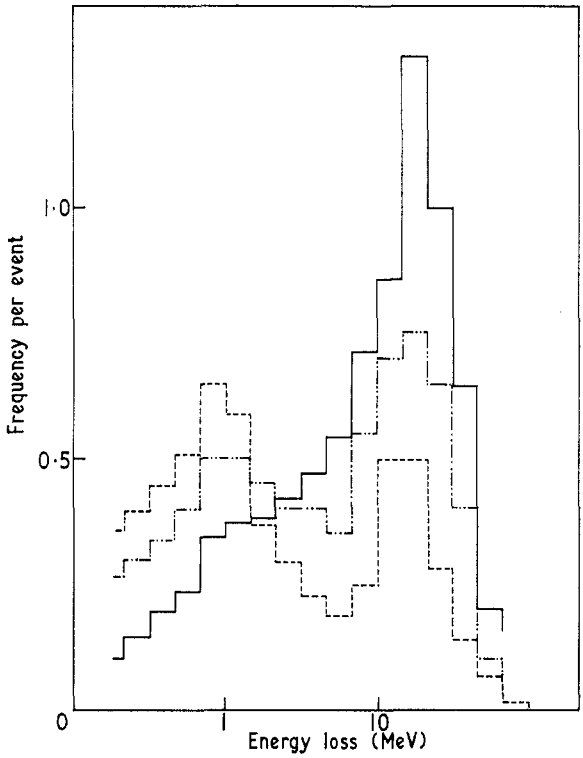

Figure 3. Scintillator pulse-height distribution. Mean core distance: —— 480 m, —··— 650 m, ------- 900 m.

Since the scintillators, because of their limited thickness, cannot differentiate between particles of energy equal to and above 16 MeV, we found it preferable to draw the pulse-height distributions, after subtraction of the muons, as integral energy spectra, and these are shown in figure 2. At the larger distances few high-energy electrons or photons are present and the spectra's steep decline well describes the energy loss of the electromagnetic component. On the other hand, in the case of the nearer core distances, the integral energy-loss spectra are not available for higher energies. Here the effect of the Landau distribution at the higher energies would in any case preclude an accurate assessment of the energy of the incident electromagnetic particles. As a result the energy-loss spectra obtained from the scintillators, while incomplete at smaller core distances, complement the data analysis at the large distances where comparison of the equivalent densities D_c and D_s breaks down because of the vanishing values of D_c.

At the smaller distances this comparison yielded flux and mean energy and hence the number N_0 of the incident electromagnetic particles, whereas at the larger distances the number N_0 and the flux is obtained, and thus the mean energy of the electromagnetic radiation, from the scintillator's energy-loss response.

6. The number of incident photons and the photon flux

Assuming a Poisson distribution the number N of interacting photons, and therefore the number N_0 of incident photons, can be estimated from the observed number of 'zeros', i.e. the number of times a scintillator does not respond to incident photons. At the larger distances the statistics of the number of zeros are sufficient (table 1) to estimate N, the number of particles which interact in the nine scintillators. Furthermore, if N is known, the average number $\bar{n}$, i.e. the multiplicity of particles interacting in the scintillators in an event, can be found. The integral spectra (figure 2) have been corrected for this multiplicity and therefore represent energy-loss spectra per interacting particle. The number N_0 of incident photons is calculated from the number N of interacting ones by allowing for the 'efficiency' of the detector which is the chance $1 - \exp(-t/\lambda)$ of an incident photon giving rise to a signal from the scintillator. Here again one has to bear in mind that λ is itself energy dependent and that the path t travelled by the particle comprises not only the thickness of the scintillator but also the layer of air above it where a photon could produce an electron capable of reaching the scintillator and causing a signal; this means that t, the total thickness, is also energy dependent.

In this way N_0 is estimated at the larger core distances where the photon energies are small and where λ decreases with energy from the near-constant value of 60 g cm^{-2}. For this estimate one needs to know at least an approximate form of the incident photon spectrum. However, this can be obtained, as outlined below, from the energy-loss spectra.

At distances greater than or equal to 800 m, the integral energy-loss spectra are sufficiently accurately known to estimate from them the energy contained in the electromagnetic component. Since $D_s - D_\mu$ is small at the largest distances and can be accurately determined only with ample statistics, the energy estimate has been confirmed with the help of the energy-loss curves (figure 2) which for the larger distances give an adequate description of the energy of the electromagnetic component.

7. The lateral distribution of the electromagnetic component

From the foregoing it is clear that by suitable combination of the data measured by the tanks and by the scintillators it is possible to cover the distances 200–1100 m from the core and to estimate in this range photon (and electron) lateral distributions and the energy carried by the electromagnetic component. The results are represented in figures 4, 5 and 6. In all figures, points obtained by comparison of equivalent densities are drawn as full circles and those estimated from the energy-loss spectra by open circles. There is seen to be good overlap at 500 m and good agreement, too, with our previous results (Dufresne *et al.* 1965) indicated by crosses. The muon data sketched in on the lateral distributions are taken from Allan *et al.* (1968), and the muon flux on figure 5 is a combination of the data of Earnshaw *et al.* (1968) and of Allan *et al.* (1968).

The proportion of the electrons at distances less than 500 m has been estimated from the density response of the scintillators, i.e. from $D_s - D_\mu$. At the larger distances it has been obtained from the ratio of the number of photons n_{ph} to that of electrons n_e, which is proportional to the ratio of their respective ranges, i.e.

$$\frac{n_{\rm ph}}{n_{\rm e}} = \frac{\lambda}{\bar{t}_{\rm e}}$$

where $\bar{t}_{\rm e}$ is the average range of the electrons. This range is energy dependent and has been obtained assuming an energy distribution for the incident photons. Yet only a very rough estimate is required to see that the number of electrons is small. In order

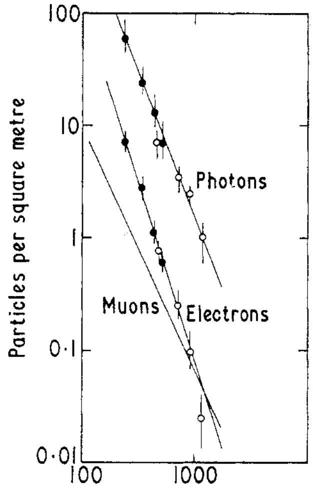

Figure 4. Lateral distribution of photons and electrons (muon data from Allan *et al.* 1968).

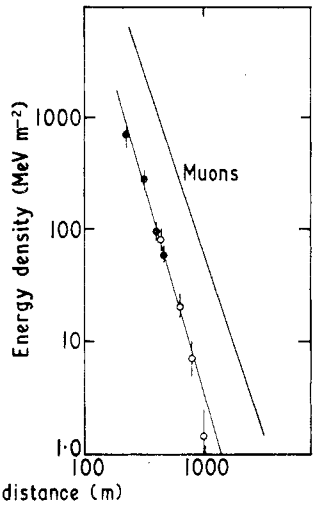

Figure 5. Energy density of electromagnetic component (muon data from Allan *et al.* 1968 and Earnshaw *et al.* 1968).

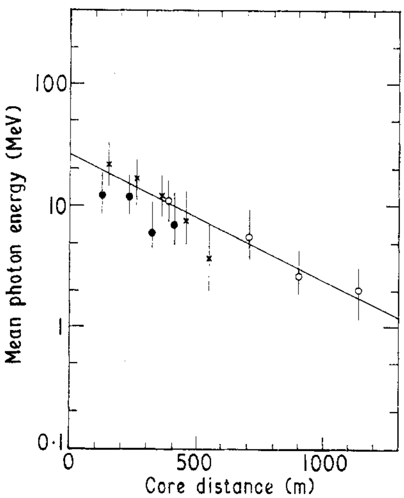

Figure 6. Variation of mean photon energy with core distance.

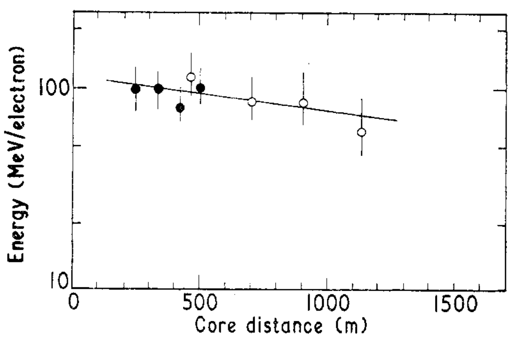

Figure 7. Energy carried by electron–photon component per incident electron.

to facilitate comparison with the results of other workers we have plotted also the 'energy per incident electron' (figure 7). This term is somewhat misleading at these large core distances where the electrons are secondary to photons, and refers in fact to the incident energy flux per observed electron.

8. The spectra of the incident photons

From the energy-loss spectra it is possible to go one step further and to arrive at an approximate estimate of the energy spectra of photons incident at some core distances. This is achieved by folding trial photon spectra into an integral expression for the energy-loss spectra. Neglecting the contribution of incident electrons, such an expression depends on the differential spectrum of photons of energy E_{ph}, $f(E_{ph})\, dE_{ph}$, the attenuation length of the photons, the fraction (itself a function of photon energy) of the photon energy given to electrons, and the effective depth of the detector. The latter includes a short distance above the detector in air in which a photon can interact and give rise to an electron seen by the scintillator. The integral then expresses the response of the scintillator to electrons produced by photons and losing an energy greater than or equal to δ to it. $f(E_{ph})$ is found by numerical fitting of the integral energy-loss spectrum. In the case of the shorter distances less than or equal to 500 m, this process can give an indication only of the number of photons of lower energy. For the larger distances the photon spectra obtained can be accepted with more

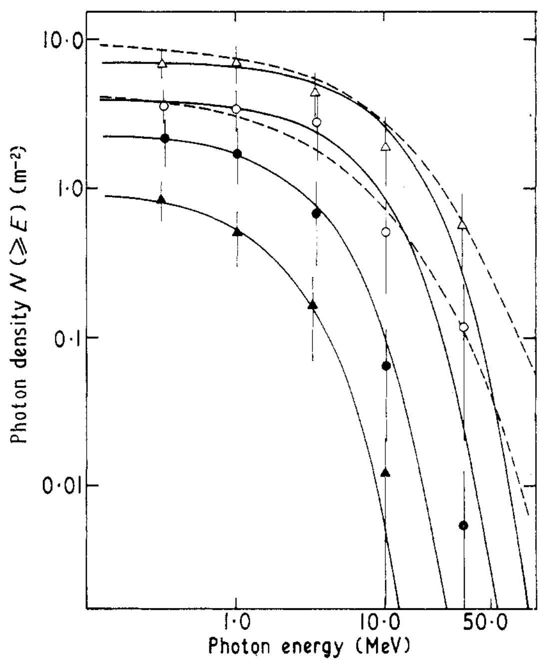

Figure 8. Integral energy spectra of incident photons. Broken line: Baxter (1969) calculated for 400 m and 600 m mean core distance. △ 480 m mean core distance; ○ 700 m mean core distance; ● 850 m mean core distance; ▲ 1000 m mean core distance.

confidence. The resulting spectra are given in figure 8 with the general form of the spectra calculated by Baxter (1969). The normalization of the theoretical curves is arbitrary and the agreement must, to some extent, be fortuitous. However, the systematic change with core distance reflects the corresponding change in the peak heights of the pulse distributions in figure 3 which was directly observed.

9. Conclusions

The plot of the energy per incident electron (figure 7), in spite of the approximations involved in its estimation, fits very satisfactorily on to the values obtained by the Tokyo group (Fukui *et al.* 1960) and at Chacaltya (Suga *et al.* 1964) and agrees, too, with our own previous estimate (Dufresne *et al.* 1965) at the distances 300–500 m. Nishimura (1967) has pointed out that nearer the core the average energy of the shower particles must be expected to remain almost constant: up to a certain minimum the mean free path of photons remains constant; the photons can travel up to a few hundred metres from the axis, whereas the electrons here are mostly secondary to the photons and can travel only a short distance. Hence the average energy of the shower particles will remain fairly constant for some hundred metres from the core. It is only at very large core distances that we must expect the energy to decrease, as is indeed borne out by figure 7.

In the case of the lateral distribution of the particles as well as of their mean energy, cascade theory can make predictions only up to about 100–200 m from the core of the showers, and it is satisfactory to see that our results fit on to those obtained by other authors for the smaller core distances. The main interest in the results here arises from the discovery of the extent to which photons are present in relatively large numbers up to large core distances and to see that they still carry an appreciable amount of energy. From the lateral distribution of numbers and of the energy flux plotted in figures 4 and 6, where we have also plotted the muon data available to us from the other Haverah Park experiments (Allan *et al.* 1968, Earnshaw *et al.* 1968), we see that even at 1000 m core distance a photon presence of 1 per square metre can be expected in these showers compared with that of 0·1 per square metre for muons. On the other hand, the energy flux carried by muons is about 10 times as large or about 100 times greater per particle.

Nearer the core, as Nishimura (1967) has shown, cascade theory would predict power laws for the lateral distribution of electrons and of their energy flux, with the latter showing a steeper slope on a logarithmic plot with an index differing by about 1 from that of the electron lateral distribution. At the distances considered, a power law of the lateral distribution of particles, which are mainly photons here, could be described approximately by an exponent $-2{\cdot}6$ compared with that of $-3{\cdot}5$ for the distribution of the energy flux.

From time to time it has been suggested that the electron–photon component at these distances could be secondary to the muon component. Our results do not bear this out: the energy flux of the electron–photon component from 1000 MeV m^{-2} to 1 MeV m^{-2} over the core distances 200–1000m must be contrasted with the corresponding energy flux for muons of 10 000 MeV m^{-2} to 50 MeV m^{-2}. It is difficult to think of any process, by Bremsstrahlung or otherwise, which could account for the relatively large energy, still carried by the electromagnetic component at the large core distances, as arising from muons. The more recent calculations by Baxter (1969) and by Hillas *et al.* (private communication) extend shower calculations to larger distances and predict lateral distributions of the electron–photon component and of its energy flux which are similar to those reported in this work. The experimental data presented here may help in the choice of suitable models for the interactions and propagation through the atmosphere of extensive air showers.

Acknowledgments

Our thanks are due to Professor J. G. Wilson for his interest in this work and to our colleagues at Haverah Park for their unfailing help when such was required. We also thank them for their comments, and in particular Dr. G. Brooke and Dr. H. W. Hunter for their criticism of the manuscript. One of us (L.T.) is indebted to the Science Research Council for a Research Studentship.

References

ALLAN, H. R., *et al.* 1960, *Proc. Phys. Soc.*, **76,** 1–16.

—— 1962, *Proc. Phys. Soc.*, **79,** 1170–82.

ALLAN, H. R., BLAKE, P. R., NEAT, K. P., and PIDCOCK, J. K., 1968, *Can. J. Phys.*, **46.** S98–101.

BAXTER, A. J., 1969, *J. Phys.* A: *Gen. Phys.*, **2,** 50–8.

DUFRESNE, R., 1968, *Ph.D thesis*, University of Leeds.

DUFRESNE, R., KELLERMANN, E. W., and TOWERS, L., 1965, *Proc. 9th Int. Conf. on Cosmic Rays, London, September* 1965, Vol. 2 (London: Institute of Physics and The Physical Society), pp.689–90.

EARNSHAW, J. C., *et al.*, 1968, *Can. J. Phys.*, **46,** S122–6.

EVANS, R. D., 1955, *The Atomic Nucleus* (New York: McGraw-Hill).

FUKUI, S., *et al.*, 1960, *Prog. theor. Phys. Suppl.*, **16,** 1–53.

GREISEN, K., 1960, *Ann. Rev. nucl. Sci.*, **10,** 63–108.

LILLICRAP, S. C., 1963, *Ph.D thesis*, University of London.

NISHIMURA, J., 1967, *Handb. Phys.*, **46/2,** 1–114.

SUGA, K., *et al.*, 1964, *Proc. 8th Int. Conf. on Cosmic Rays, Jaipur*, 1963, Vol. 4 (Bombay: Commercial Printing Press), pp. 9–23.

TENNENT, R. M., 1967, *Proc. Phys. Soc.*, **92,** 622–31.

VERNOV, S. N., *et al.*, 1968, *Can. J. Phys.*, **46,** S197–200.